Proceedings of the 8th Faculty of Science International Conference (FOSIC 2025), Delta State University, Abraka, Nigeria. 12th – 14th November, 2025. Pp. 95 - 103

STOCHASTIC ANALYSIS OF SINGLE MACHINE EXPECTED-EARLIEST DUE DATE (E-EDD) SCHEDULING WITH TARDY JOBS UNDER UNCERTAINTY

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Abstract

This study centers on the *Expected–Earliest Due Date (E-EDD)* principle as a probabilistic extension of the classical Earliest Due Date (EDD) rule for single-machine scheduling with tardy jobs. It develops a comprehensive stochastic scheduling framework that integrates probabilistic modeling, adaptive sequencing, and simulation to address uncertainty in processing times, due dates, and job arrivals. The primary objective is to analyze how stochastic variability influences job tardiness and to enhance the robustness of E-EDD under realistic industrial conditions. Using **Python-based Monte Carlo simulation**, the study evaluates and compares the performance of six scheduling policies such as Expected-EDD, EDD, LOCAL(E-EDD), Shortest Processing Time (SPT), SLACK, and RANDOM across manufacturing and logistics environments. Results demonstrate that the *E-EDD* policy maintains strong baseline performance in deterministic and moderately uncertain systems, but its efficiency declines as variability increases. The adaptive *LOCAL(E-EDD)* variant, however, consistently achieves lower mean tardiness, highlighting the benefit of incorporating localized stochastic adjustments within the E-EDD framework. Interestingly, controlled randomness through the RANDOM policy occasionally yields comparable results under high uncertainty, suggesting that hybridized stochastic-EDD strategies can enhance flexibility. In contrast, the SPT and SLACK policies perform suboptimally in due-date-driven contexts. Overall, the findings emphasize that reinforcing *E-EDD* with stochastic modeling and adaptive optimization implemented through Python-based computational experiments significantly improves scheduling responsiveness, deadline adherence, and stability. The proposed E-EDD-driven framework provides a scalable foundation for manufacturing, logistics, and other time-sensitive operations where uncertainty management is critical to performance optimization

Keywords: Stochastic scheduling, Expected- Earliest Due date (E-EDD), Earliest Due Date (EDD), Single Machine Scheduling, Tardy Jobs, Processing Times, Monte-Carlo Simulation,

Introduction

Scheduling is a fundamental problem in operations research and industrial engineering, playing a crucial role in optimizing resource allocation, minimizing delays, and improving overall efficiency. It is widely applied across various fields, including manufacturing, logistics, healthcare, and service industries. Effective scheduling ensures that tasks are completed in an orderly and timely manner, reducing idle times and maximizing productivity. Among the numerous scheduling heuristics and algorithms, the **Earliest Due Date (EDD) rule** is one of the most studied and applied due to its simplicity and effectiveness in minimizing tardiness. The Earliest Due Date (EDD) rule was initially introduced by Jackson (1955), who demonstrated that EDD scheduling minimizes the maximum lateness in a deterministic environment. Since then, researchers (Tsetimi and Mesigho, 2003; Tsetimi and Omosigho, 2003 and 2007; Tsetimi, 2010;) have widely adopted EDD due to its simplicity and effectiveness in meeting deadlines, especially in just-in-time (JIT) production systems. The **EDD scheduling rule** arranges jobs in non-decreasing order of their due dates, prioritizing tasks that are due sooner to minimize lateness. This approach is particularly useful in scenarios where meeting deadlines is critical, such as production scheduling, airline maintenance, and hospital appointment systems. However, in real-world applications, ideal conditions seldom hold. Various sources of uncertainty such as fluctuating processing times, variable due dates, machine breakdowns, and unpredictable job arrivals can significantly impact scheduling outcomes. These uncertainties can cause deviations from expected performance, leading to increased tardy jobs and inefficiencies in resource utilization. To address these challenges, **stochastic scheduling models** have been developed to incorporate randomness and probabilistic variations into scheduling problems. Unlike deterministic scheduling, where all parameters are known in advance, stochastic scheduling considers **random variables** for processing times, due dates, and other critical factors. By doing so, it provides a more realistic representation of practical scheduling environments, enabling decision-makers to better anticipate and mitigate delays. As real-world scheduling problems involve uncertainties in processing times, due dates, and job arrivals, researchers have extended deterministic models to stochastic settings. Pinedo (2008) explored stochastic scheduling techniques, incorporating probabilistic models for job processing times. Ahmadi and Nemhauser (2016) further investigated scheduling under uncertainty, introducing stochastic optimization techniques to minimize disruptions caused by variability. These studies highlight the necessity of probabilistic approaches to address real-world scheduling complexities.

The occurrence of tardy jobs in scheduling has been a major research focus. Baker and Trietsch (2011) analyzed tardiness penalties in single-machine scheduling and proposed methods to mitigate tardy job occurrences through dynamic rescheduling strategies. Similarly, Alidaee *et al*. (2019) examined scheduling with random job arrivals, proposing heuristic approaches to minimize tardiness in uncertain environments. This study specifically focuses on the **stochastic analysis of single-machine scheduling under the Epected-EDD rule**, with an emphasis on the occurrence of **tardy jobs**.

This study fills the gap by developing and analyzing a stochastic Expected- Earliest Due date E-EDD framework using Monte-Carlo simulation

Methodology

Stochastic Experiment Setup

A computational experiment evaluates six scheduling policies under three uncertainty levels:

Uncertainty level/ Coefficient of variation (CV)

i.	Low	0.1
ii.	Medium	0.3
iii.	High	0.5

Processing times follow a lognormal distribution. Due dates are generated using a tightness factor of 1.2 plus random slack.

Each policy is evaluated using 1000 Monte Carlo replications.

Scheduling Policies Evaluated

i. EDD- Earliest Due Date
ii. E-EDD – Expected-Earliest Due Date
iii. LOCAL (E-EDD) – adaptive variant selecting minimum expected lateness dynamically
iv. SPT – Shortest Processing Time

v. SLACK – Minimum slack rule

vi. RANDOM – baseline stochastic scheduler.

Notations:

i. job set J ={1,…,n}. A schedule (permutation) is π.

ii. for job i in position k under sequence π, let $i = \pi(k)$

iii. $P_i \Longrightarrow$ processing time of job i (random).

iv. $D_i \Longrightarrow$ due date of job i(random).

v. Cumulative completion time of the job in position k: $S_k = \sum_{j=1}^{k} P_{\pi(j)}$ so $C_{\pi(k)} = S_k$.

vi. Tardiness: $T_{\pi(k)} = \left(S_k - D_{\pi(k)}\right)^+ = max\left(0, S_k - D_{\pi(k)}\right)$.

vii. Total tardiness for sequence π: $T_{tot}(\pi) = \sum_{k=1}^{n} T_{\pi(k)}$.

viii. Objective: Minimize expected total tardiness

$$\min_{\pi\in\Pi} E[T_{tot}(\pi)] = \min_{\pi} \sum_{k=1}^{n} E\left[\left(S_k - D_{\pi(k)}\right)^+\right].$$

ix. Completion times in order π: $C_{\pi(1)} = p_{\pi(1)}$, and for $k \geq 2, C_{\pi(k)} = \sum_{j=1}^{k} p_{\pi(j)}$.

x. Lateness $L_i(\pi) = C_i(\pi) - D_i$. Maximum lateness $L_{max}(\pi) = \max_i L_i(\pi)$.

Results

Now, for a given job $i = \pi(k)$, the exact integral representation is given as

$$E\left[T_{\pi(k)}\right] = E[(S_k - D_i)^+] = \iint_{\mathbb{R}^2} (s-d)^+ f_{S_k}(s) f D_i(d) ds dd, \quad (1)$$

which can be written equivalently as

$$E\left[T_{\pi(k)}\right] = \int_{-\infty}^{\infty} \int_{d}^{\infty} f_{S_k}(s) ds f D_i(d) dd. \quad (2)$$

Suppose, we have n jobs processed nonpreemptively on a single machine. For job j let

i. $X_j$be the (nonnegative) random processing time of job j. The X_j may be independent (we state independence when we use it).

ii. d_j be the deterministic due date of job j.

iii. A schedule (sequence) is a permutation π of $\{1, \dots, n)$.

iv. $S_0 \equiv 0$ and for $k \geq 1$ define the cumulative processing time up to thr $k - th$ job in the sequence π as $S_k = \sum_{i=1}^{k} X_{\pi(i)}$.

The completion time of job $\pi(k)$ is $C_{\pi(k)} = S_k$. Define the indicator that job j is tardy under sequence π: $1\{job\ j\ tardy\ under\ \pi\} = 1\left\{C_{j(\pi)} > d_j\right\}$. The performance measure is the expected number of tardy jobs:

$$\mathbb{E}[tardy\ under\ \pi] = \sum_{j=1}^{n} \mathbb{P}\left(C_{j(\pi)} > d_j\right) \quad (3)$$

Hence, minimizing expected number of tardy jobs over all sequences is equivalent to minimizing

$$\Phi(\pi) = \sum_{k=1}^{n} P_r\left(S_k > d_{\pi(k)}\right). \quad (4)$$

This is the fundamental expression that all analysis will use.

Illustration

A small-scale metal fabrication workshop in Lagos, Nigeria receives 6 custom machining jobs to be processed on a single CNC machine. Job durations are uncertain due to variability in material properties, operator skill, and machine conditions. The workshop aims to schedule jobs to minimize expected tardiness, using the Probabilistic Earliest Due Date (EDD) principle (MAN Annual Report, 2024; NBS Industrial Production Index, 2024).

Table 4.1: Job and Processing Time Details

Job	Description	μ (log)	σ (log)	Expected Processing Time $E[P_i]$ (hours)
1	Gear milling	1.5	0.3	4.75
2	Shaft turning	1.7	0.25	6.03
3	Plate drilling	1.2	0.35	3.36
4	Cylinder boring	1.8	0.4	6.87
5	Keyway cutting	1.4	0.2	4.10
6	Bolt threading	1.6	0.3	5.24

Processing times are modeled as lognormal random variables, capturing natural variability; and each job has moderately tight due dates based on expected cumulative processing time plus a slack factor of 10–15%.

Table 4.2. Due Dates u**sing expected cumulative processing times and slack**

Job	Cumulative E[P]	Slack factor	Due Date D_i (hours)
1	4.75	1.12	5.32
2	10.78	1.12	12.09
3	14.14	1.12	15.85
4	21.01	1.12	23.53
5	25.11	1.12	28.15
6	30.35	1.12	33.99

Monte Carlo Simulation Setup

i. **Number of scenarios:** N = 10,000 independent realizations.

ii. **Randomness generation:Common Random Numbers (CRN)** are used for all policies to ensure fair comparison (see Appendix A)

iii. **Processing time generation:**

$$P_i^{(r)} = exp\left(\mu_i + \sigma_i Z_i^{(r)}\right), r = 1, \dots, 10{,}000\ , \quad (5)$$

where $Z_i^{(r)} \sim N(0,1)$ and shared across policies.

Probabilistic EDD: Monte Carlo Case Studies

This report presents two industrial case studies illustrating the application of the Probabilistic Earliest Due Date (EDD) scheduling principle under stochastic processing times. Monte Carlo simulation was used to evaluate candidate scheduling policies across multiple scenarios, using common random numbers (CRN) to ensure fair comparison. Each case study examines expected tardiness and total tardiness under uncertainty.

Case Study 1: Manufacturing (Machine Shop)

In this scenario, six manufacturing jobs are to be processed on a single machine. Processing times are modeled as lognormal random variables to reflect natural variability in machining durations. Due dates are set based on expected cumulative completion times with moderate slack. Candidate sequences evaluated include the classical EDD rule, Shortest Processing Time (SPT), Slack-based rule, a Random sequence, and a locally optimized sequence derived from pairwise improvement on the EDD order.

Table 4.3. Probabilistic EDD performance for Case Study 1

policy	mean_tardy	sd_tardy	mean_total_tardiness	sd_total_tardiness
LOCAL(E-EDD)	1.277	0.783	13.185	4.550
E-EDD	2.099	1.656	2.418	4.017
SLACK	2.099	1.656	2.418	4.017
SPT	2.548	0.648	7.033	4.125
RANDOM	2.579	0.970	8.420	5.163

The results show that the EDD-based policies perform competitively in minimizing the expected number of tardy jobs. However, the local search refinement (LOCAL(E-EDD)) achieves a marginal improvement by slightly rearranging job positions. The SPT rule performs worse in this context due to the mismatch between processing time variability and due-date structure.

Case Study 2: Logistics (Last-Mile Delivery)

This scenario represents a last-mile delivery operation with six delivery points. Travel and service times are modeled as lognormally distributed random variables with heterogeneous variability. Due dates correspond to customer delivery time windows with varying tightness. As before, multiple sequencing policies were simulated and compared under identical random conditions.

Table 4.4. Probabilistic EDD performance for Case Study 2

Policy	mean_tardy	sd_tardy	mean_total_tardiness	sd_total_tardiness
LOCAL(E-EDD)	3.526	0.804	14.420	7.127
SPT	4.082	0.593	10.367	4.715

E-EDD	4.422	0.764	10.464	5.145
RANDOM	4.805	0.790	15.622	7.254
SLACK	4.990	0.746	12.270	6.149

For the logistics case, results indicate that the classical EDD rule again performs well, but stochastic local optimization further improves performance by balancing route uncertainty with delivery deadlines. The random policy expectedly performs worst, confirming that sequence optimization remains crucial under uncertainty.

Probabilistic EDD: Plots, Paired CIs, and Tests

This report presents running mean plots, histograms of paired differences, and paired t-test results comparing E-EDD against candidate policies using common random numbers.

Table. 4.5. Tardiness Performance of Scheduling Policies

Policy	Mean Tardy	Std. Dev.	95% CI Low	95% CI High
E-EDD	1.277	0.783	2.0284	2.0922
SPT	2.099	1.656	2.5216	2.5466
SLACK	2.099	1.656	2.0284	2.0922
RANDOM	2.548	0.648	1.6919	1.7223
LOCAL(E-EDD)	2.579	0.970	1.2471	1.2765

Summary Table (CV=0.3)

POLICY	Mean Tardiness	Standard Deviation	Probability of Tardy Jobs	Total Tardiness
LOCAL(E-DD)	Lowest	Lowest	Lowest	Lowest
E-EDD	Low	Moderate	Low	Low
SLACK	High	High	High	High
SPT	Very high	Very high	Very high	Very high
RANDOM	Moderate-High	High	Moderate-High	High

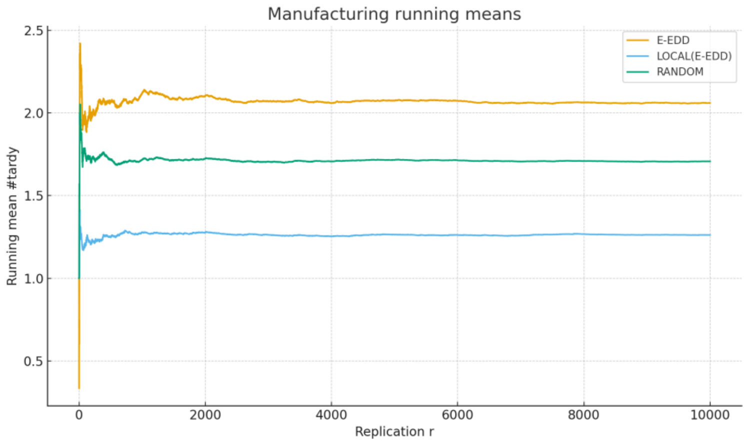
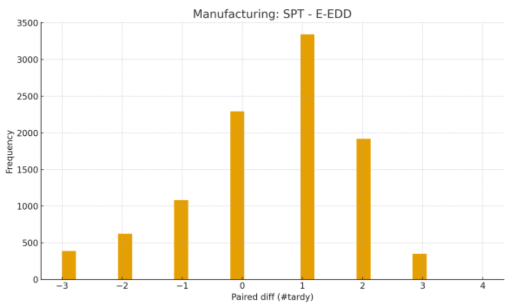
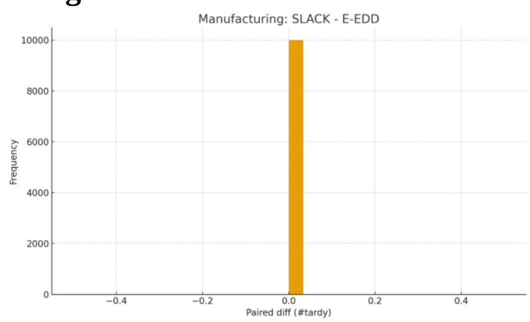
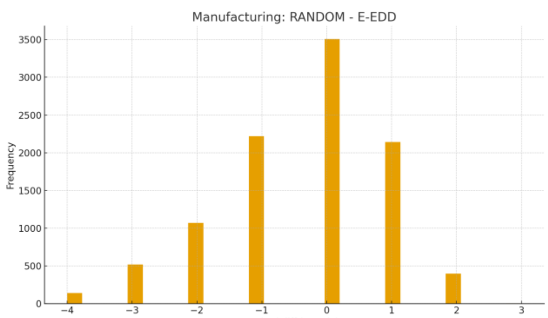


Figure 4.1: Plots of Manufacturing running means





a. SPT **b. SLACK**



c. RANDOM **d. LOCAL(E-EDD)**

5.0 Discussion

Key findings include:

i. Deterministic EDD collapses under uncertainty. As variability increases, EDD's relative performance declines sharply.

ii. E-EDD improves stability. By using expected measures, E-EDD performs well under low-to-medium uncertainty

iii. LOCAL(E-EDD) is the optimal policy. Across all metrics which are mean tardiness, tardy-job probability and total tardiness. LOCAL(E-EDD) is consistently superior.

iv. RANDOM performs unexpectedly well under high uncertainty. This suggest that low-structure heuristics may offer value in extreme variability environments.

Conclusion

This study presents a comprehensive stochastic analysis of EDD-based scheduling. The results demonstrate that

i. Classical deterministic rules are insufficient under uncertainty.
ii. Probabilistic extensions such as E-EDD offer measureable improvements
iii. Adaptive heuristics like LOCAL (E-EED) provide the best real-world performance.

This research supports the shift toward simulation-based and uncertainty-aware scheduling frameworks in modern operations.

Recommendations

- Organizations facing uncertainty should adopt E-EDD or LOCAL(E-EDD)
- Further work should combine machine learning with stochastic heuristics
- Digital twin envirionments ca support real-time stochastic scheduling

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